AC’s Method of One-line Boolean Symmetry Detection

Anirban Chakraborty
Assistant Professor, Barrackpore Rastraguru Surendranath College, Department of Computer Science, Barrackpore, Kolkata -120, West Bengal, India

Abstract - The detection of symmetries in a switching function is an NP-complete problem. In this paper a single line solution for detecting total symmetry been proposed. The proposed method exploits the linearity among Boolean terms to obtain the unique transformation matrix associated with every linear function. A theorem has been proved which relates the equality of coefficients in the transformation matrix, of the variables with symmetry detection. By using the proposed methodology, the simplicity for detecting total symmetries been improved in terms of cost in time-space domain.

Keywords: Symmetry Detection, Switching Function, Transformation Matrix, Linear Function.

I. INTRODUCTION

Identification of a symmetric function is an interesting problem, because a function can exhibit symmetry in all possible polarities of its input literals. More precisely, to detect the symmetry of an n variable switching function, one needs to explore 2n possible literals of the input variable’s permutation. It is essential to know when a function is totally or partially symmetric, as there exist different standard synthesis procedures for integrated circuits that realize such functions [11]. Moreover, symmetry detection has a very important role in complex circuit design. Identification of symmetries in switching literals can aid in efficient, robust and economic logic synthesis process. Different existing models work tirelessly to achieve much sought after detection of symmetries; as it easies circuit design, reduces manufacturing cost among other advantages, which makes its use and knowledge immensely valuable and yet it remains hard and unknown to many. All of these existing methods employ maps, tables, charts etc. where space-time is concerned. The labyrinthine world of symmetries has attracted the attention of countless mathematicians each taking a shot at their very own detection method. Here a simple single equation (Mono-line) method for detection of total symmetry been proposed. As the function is linear one, an associated transformation matrix also exists, which could further be analyzed for detection of symmetry based on equality criterion. Symmetries in literal are equal as the respective elements of the transformation matrix and this property has been used for detection.

II. RELATED WORKS

Symmetry detection has always been an area of interest for researchers. It has been well studied and used in many applications, such as functional decomposition in technology independent logic synthesis, technology mapping, BDD minimization, testing and verification. Several algorithms have been proposed to detect classical symmetries more efficiently. Different methods exist to detect symmetry in switching functions. A convenient way of writing a symmetric function is obtained from the theorem of Shannon [13]. S.H. Caldwell [15] has described a method for identifying total symmetry of switching functions with the help of extended Karnaugh maps, though it is difficult for functions of larger than four variables. The identification method proposed by Marcus [17] and Mc Cluskey [16] are more systematic and applicable for large number of variables. A. Mukhopadhyay [8] formulated a technique for detecting symmetry using a chart based method; popularly known as decomposition chart. S. R. Das and C. L. Sheng [9] used an approach known as residue test by numerical method to detect symmetry. Tsai et al. [15] used generalized Reed-Muller forms for detecting groups of symmetric literals in completely specified switching functions. Kravets and Sakallah [22], and J.S. Zhang et al. [26], both used circuit based method that makes uses of structural analysis, integrated simulation and Boolean SAT for fast and scalable detection of symmetries switching function. Rice and Muzio [3] detailed in minimizing the BDDs. BDDs also return the favour helping to detect symmetries and anti-symmetries. Kettle et al. [27] found generalized symmetries in binary decision diagram (BDD). Z. Kohavi [10] introduced an approach for finding symmetric switching functions based on invariant under any permutation of literals. S. Guha et al. [25] modified the method of Z. Kohavi where the true terms are bi-partitioned into two subsets depending upon the polarity of a particular literal to reduce the search space. All of these methods employ maps, truth tables, charts, matrices etc. Picek et al. [4] have made attempts to visualize the symmetry finding problem as a search problem as well as an optimization problem, called fighting the symmetry. Heuristic search methods were used to find optimal results for more than 4-6 variable problems, where the symmetry structure is extended with some observations on fitness landscape to convert the problem into optimality finding problem, the search approach had certain disadvantages as pointed out by
Sebastian et al. whereas using the natural structure of the problem is easier to analyze. Boyd et al. [1] visualized symmetry detection process with Markov process as a graph of permutations of inputs and prunes it till a manageable sub graph remains which could be searched for symmetry. They use concepts from group theory to show that if any two symmetries exist then applying group operations upon them will discover a new pair of symmetry. To use an AND Inverter Graph (AIG) as an input to their algorithm, which constructs a permutation tree pruning it by using group theoretic approaches. The algorithm iteratively refines the search space, it is also smart enough to recognize equivalence and avoid recurring conflicts. Mearsand et al. [12] consider the problem as a constraint satisfaction problem (CSP) and solved for to detect symmetry use AIG as input and combine it with Boolean Satisfiability problem, to iterate through for a solution. Where they create a concurrent model which converts the problem to nondeterministic sequential model. The worst case complexity of such a model is linear. They have envisioned a model to find similarity in symmetry and constraint programming. Transforming the sequential model into constraint satisfaction problem and making CSP as a graph called colored graph, considering each automation of the colored graph as one in the concurrent model which was in-turn discovered and named ‘Saucy’ as a graph automorphism generator. Analogous to the working and existence of different methods whose sole reason for existence is ease of introduction and use, makes them viable in this day and age; where certainly far better and faster algorithms exist; where certainly far better and faster algorithms exist. In this paper the symmetries been visualized algebraically on the linearity among functions to obtain the unique transformation matrix. Further it co-relates with the equality of coefficients in the matrix to identify symmetries.

III. PRELIMINARIES

Definition 1.[11] A switching function $f$ of $m$ variables $\{x_{m-1}, x_{m-2}, \ldots, x_0\}$ is said to be totally symmetric iff any permutation of these $n$ variables leaves the function invariant. Therefore, any permutation of variables may be obtained by successively interchanging only two variables at one time.

Example: $f(x_2, x_1, x_0) = \bar{x}_2\bar{x}_1x_0 + x_2\bar{x}_1\bar{x}_0 + \bar{x}_2x_1\bar{x}_0$ is totally symmetric because any permutation of these 3 variables leaves the function invariant.

Conjecture 1.[11] By definition of total symmetry of switching functions, it can be conclude that is $x_i$ invariant $x_j$ .i.e., $x_i$ and $x_j$ can be interchanged without altering the outcome of the function: $\forall i = 0$ to $m-1$ and $\forall j = 0$ to $m-1$ and $i \neq j$, (i.e., for all permutation of $x_i$ and $x_j$) then the function is totally symmetric.

A symmetric function consists of mixed variables of symmetry in general, if some of the variables are unprimed and rest of them are primed. The variables of symmetry are said to be multiform in nature when the function exhibits multiform symmetry in these variables.

Theorem 1 [10]: The number of total symmetric functions of $m$ variables is $2^{(m+1)}$ where $m$ is the number of literals.

Definition 2. [27] The transformation matrix $\theta$ be a matrix in $R$ vector space and let $T: R^n \rightarrow R^n$ defined by $T(x) = \theta x$ be the associated matrix transformation. The domain of $T$ is $R^n$, where $m$ is the number of columns literals. The co-domain of $T$ is $R^n$, where $n$ is the number of rows of $\theta$. The range of $T$ is the column space of $\theta$.

IV. PROPOSED THEOREM

Let $Y$ be a function containing $m$ input variables and $n$ symmetric variables. Let $a$ be an array of size $n$ containing all the symmetric variables of $f$, in order in which they are encountered from left to right in $Y$. Hence $a[0]$ contains the first of the symmetric variables encountered from left to right in $f$, and similarly, $a[n-1]$ contains the $n^{th}$ symmetric variable from left to right in $Y$. Order doesn’t matter for the symmetric variables of $Y$, however they are placed in order, just for notational convenience.

Assumptions, $Y$ is linear. Hence there exists a transformation matrix $\theta$, associated with $Y$, such that

$$f(x_0, x_1, \ldots, x_{m-1}) = c_0 x_0 \oplus c_1 x_1 \oplus \ldots \oplus c_{m-1} x_{m-1},$$

where $c_0 \ldots c_{m-1}$ is the co-efficient.

4.1 Proposed Theorems

Theorem 2: (Tandem Duo Swap Theorem)

$$c_{a[0]} x_{a[0]} \oplus c_{a[1]} x_{a[1]} \oplus \ldots \oplus c_{a[n-1]} x_{a[n-1]} = c_{a[0]} x_{a[0]} \oplus c_{a[1]} x_{a[1]} \oplus \ldots \oplus c_{a[n-1]} x_{a[n-1]} x_{a[n-2]}$$

Proof:

Let, $f(x_0, x_1, \ldots, x_{a[n-1]})$ be the function having symmetry on variables $x_{a[0]}, x_{a[1]}, \ldots, x_{a[n-1]}$ where $n$ is the number of variables having symmetry among them in $f$. 

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As \( x_{0(0)}x_{1(1)}...x_{0(n-1)} \) are symmetric in \( f \), any permutation of leaves \( f \) remains the same.

Hence, \( f(*,x_{0(0)},*,x_{1(1)},*,...*,x_{0(n-2)},*,x_{0(n-1)},*) = f(*,x_{0(0)},*,x_{1(1)},*,...*,x_{0(n-2)},*)...(*) \)

Let \( \theta = \{a_0 c_1 ... c_m \} \) be the transformation matrix associated with \( f \). As \( f \) doesn't change with the permutation of the symmetric variables neither does \( \theta \).

\[
f(x_0,x_1,...,x_m) = c_0 x_0 \oplus c_1 x_1 \oplus ... \oplus c_m x_m \)

Now,

\[
f(x_0,x_1,...,x_m) = \bigoplus_{i=0}^n c_i x_i \]

From (i),

\[
f(*,x_{0(0)},*,x_{1(1)},*,...*,x_{0(n-1)},*) = f(*,x_{0(0)},*,x_{1(1)},*,...*,x_{0(n-2)},*)...(*) \]

by using (ii) and (iv),

\[
c_{0(0)}x_{0(0)} \oplus c_{1(1)}x_{1(1)} \oplus ... \oplus c_{0(n-2)}x_{0(n-2)} \oplus ... \oplus c_{0(n-1)}x_{0(n-1)} \]

As \( A \oplus X = B \), \( A = B \)

Hence, \( c_{0(0)}x_{0(0)} \oplus ... \oplus c_{0(n-1)}x_{0(n-1)} = c_{0(0)}x_{0(0)} \oplus ... \oplus c_{0(n-2)}x_{0(n-2)}x_{0(n-1)} \).

\textbf{Theorem 3: (Coefficient Equality Theorem)}

If \( x_{0(0)},x_{1(1)},... ,x_{0(n-1)} \) are symmetric variables of a switching function \( Y \), then \( c_{0(0)}c_{1(1)}...c_{0(n-1)} \) the coefficients of the transformation matrix associated with them are equal, i.e.,

\( c_{0(0)} = c_{1(1)} = ... = c_{0(n-1)} \).

\textbf{Proof:}

By induction, the following statement can be proved.

\[
c_{0(0)}x_{0(0)} \oplus c_{1(1)}x_{1(1)} \oplus ... \oplus c_{0(n-2)}x_{0(n-2)}x_{0(n-1)} = c_{0(0)}x_{0(0)} \oplus c_{1(1)}x_{1(1)} \oplus ... \oplus c_{0(n-2)}x_{0(n-2)}x_{0(n-1)} \]

If only two variables \( x_i \) and \( x_j \) of \( f \) are symmetric, i.e.,

\[
f(*,x_i*,x_j*) = f(*,x_j*,x_i*) \]

or,

\[
c_{x_i} \oplus c_{x_j} \oplus ... \oplus c_{x_{j-1}}x_k \]

as \( A \oplus X = B \), \( A = B \),

\[
c_{x_i} \oplus c_{x_j} = c_{x_j} \oplus c_{x_i} \]

Hence, the Base case holds.

Let, \( c_{0(0)}x_{0(0)} \oplus c_{1(1)}x_{1(1)} \oplus ... \oplus c_{0(n-1)}x_{0(n-1)} = c_{0(0)}x_{0(0)} \oplus c_{1(1)}x_{1(1)} \oplus ... \oplus c_{0(n-2)}x_{0(n-2)}x_{0(n-1)} \) be true.

Thus we have to show, \( c_{0(0)}x_{0(0)} \oplus c_{1(1)}x_{1(1)} \oplus ... \oplus c_{0(n-2)}x_{0(n-2)}x_{0(n-1)} \) is true.

\[
LHS = c_{0(0)}x_{0(0)} \oplus c_{1(1)}x_{1(1)} \oplus ... \oplus c_{0(n-2)}x_{0(n-2)}x_{0(n-1)} \oplus c_{0(n-1)}x_{0(n-1)}
\]

by induction hypothesis.

By using Theorem 2,

\[
LHS = c_{0(0)}x_{0(0)} \oplus c_{1(1)}x_{1(1)} \oplus ... \oplus c_{0(n-2)}x_{0(n-2)}x_{0(n-1)} \oplus c_{0(n-1)}x_{0(n-1)} \]

\( \Rightarrow \)

\( RHS \)

This proves the induction hypothesis.

Hence, \( c_{0(0)}x_{0(0)} \oplus c_{1(1)}x_{1(1)} \oplus ... \oplus c_{0(n-2)}x_{0(n-2)}x_{0(n-1)} \)

Or,

\( c_{0(0)}x_{0(0)} \oplus c_{1(1)}x_{1(1)} \oplus ... \oplus c_{0(n-2)}x_{0(n-2)}x_{0(n-1)} \)

Therefore, \( (c_{0(0)}c_{1(1)}...c_{0(n-1)}) (x_{0(0)} \oplus x_{1(1)} \oplus ... \oplus x_{0(n-2)}x_{0(n-1)} \)

\( = (c_{0(0)}c_{1(1)}...c_{0(n-1)}x_{0(0)} \oplus x_{1(1)} \oplus ... \oplus x_{0(n-2)}x_{0(n-1)} \)

Let \( \theta_1 = (c_{0(0)}c_{1(1)}...c_{0(n-1)}) \),

\( \theta_2 = (c_{0(0)}c_{1(1)}...c_{0(n-1)}) \),

\( X = (x_{0(0)}x_{1(1)}x_{2(2)}...x_{0(n-2)}x_{0(n-1)}) \)

Therefore, \( \theta_1^TX = \theta_2^TX \), from matrix transformation rule

\( \Rightarrow \)

\( \theta_1 = \theta_2 \)

Hence proved, coefficients of symmetric variables of a function are equal.
4.2 Procedure: Finding Symmetric Groups

**Input:** A switching function $Y$ of $n$ variables.

**Output:** Symmetric variable groups.

1. Arrange the input and output of $Y$ as per the Transformation Matrix rule.
2. Obtain $\theta$ from the Transformation Matrix.
3. According to Coefficient Equality Theorem associated variables of equal valued elements of $\theta$ are Symmetric.

Example: Following is an R code demonstrating the algorithm:

```r
x = rbind(c(0,0,0,0,0,0,0),c(0,0,0,0,0,0,1),...,c(1,1,1,1,1,1,0),c(1,1,1,1,1,1,1))

y=rbind(1,1,0,0,0,0,0,0,1,0,0,0,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0)

theta=t(inv(t(x)%*%x)%*%t(x)%*%y)
cat(theta)
```

Output:

```
-0.01171875 0.01953125 0.01953125 0.05078125 0.01953125 0.01953125
```

If the variables were ABCDEFG in that order, then from the transformation matrix we can conclude BCEFG are symmetric. (by using Coefficient Equality Theorem).

V. EXPERIMENTAL RESULTS

All the tests have been performed on Windows 10 enterprise version 1803, implemented in R and Python programming language. Standard approaches like decomposition chart [8], residue [9], Kohavi [10] and parametric based method [11]been taken for performance analysis and comparison.

Figure 1 depicts that the proposed method works well than the other considered.

Figure 2: Average time analysis for large number of terms

Further, the experimental facts been performed up to 100 terms of fixed polarity of literals as random test cases, shows in figure 2. The Fig. 2 depicts the performance line trend to exponential as the terms grow in size; though the mono-line method performs well with respect to other methods.

Figure 3: Switching time comparison with LGSynth91 benchmark

Figure 3 represents the switching time comparisons over LGSynth91 [26] benchmark, Fig. 3 shows that it looks good except in my_adder and mux. In overall scenario the proposed method been improved in average cases.

VI. CONCLUSIONS

Symmetry detection is an NP-complete problem. As the area size (product terms) of complex circuits increase, the existing symmetry detection algorithms require faces combinatorial explosion. Here a single equation (Mono-line) method for detection of total symmetry been proposed. The comparison charts show the increased amount of time with the number of literal grows in different methods, where the proposed method performs well in average scenario. At a few glances the mono-line method exhibits higher amount of time which is due to the inert behavior of matrix operations. Otherwise it proves to be a simple one line equation model which can be used as a quick fix tool for totally symmetry detection in existing state of the art technology. The attempt is
worth studying, the universal symmetry detection is our own envy. Our next venture would be developing an effective data model for large number of terms, which may lead towards constructing the universal logic for all categories of symmetry.

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