Use the Auxiliary Projection Method to Solve Problems in Geometry

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Abstract - Geometry is a science with very wide applications in life. Everything around us is geometric shapes. Solving geometry problems helps us solve many real-life problems. In geometry there are many complex problems. But if a solution is found, many difficult problems will be solved quickly and easily. This article presents the auxiliary projection method to solve many problems in geometry. The following data can be determined.

Keywords: Bisecting plane 1: Pg1, Bisecting plane 2: Pg2, Front plane of projection: π1, Top plane of projection: π2.

I. INTRODUCTION

Solved by projecting the image onto the projection plane and solving the problem on its projection. Then from the results on the projection we will infer the results in space.

This method demonstrates how to select a new projection direction and a new projection plane and project the image onto the new projection plane. Solve the problems with new projections. Then convert this result to the old projection.

In addition to the basic projection planes, we can choose bisector planes.

The points are on two bisector planes are equidistant from the π1 and the π2.

II. APPLICATION TO SOLVE SOME GEOMETRIC PROBLEMS

Problem 1: Draw the intersection of the plane ABC and profile line EF.

We project both plane ABC and EF onto the Pg2. The new projection direction is AB. The new projection of plane ABC is a line. This line intersects line EF at point H'. We project H' onto the π1 and the π2 we obtained H1 and H2. (See fig 4)
**Problem 2:** Draw the intersection of two figure planes ABC and DEF.

We project both plane ABC and DEF onto the $\Pi_2$. The new projection direction is EF. The new projection of plane DEF is a line. The new projection of the intersection of the two figure planes coincides with this line. Projecting this intersection line in opposite directions onto the $\Pi_1$ and the $\Pi_2$ we obtained two straight lines $G_1H_1$ and $G_2H_2$. These are two projections of the intersection of two given figure planes (See fig 5).

**Problem 3:** Draw the intersection of line d and the prism ABCA'B'C' (See figure 6).

We project both the prism ABCA'B'C' and line d onto the $\Pi_2$. The new projection direction is AA'. The new projection of the prism is a triangle. The new projection of line d is line $d'$. We will find the intersection of line d with the prism on the new projection as two points $E'$ and $F'$. Projecting $E'$ and $F'$ in opposite directions onto the $\Pi_1$ and the $\Pi_2$ we obtained $E_1F_1$ and $E_2F_2$.

**Problem 4:** Given two skew lines b and d. Find points on line d and a distance r from line b.

The points to be found will lie on a right circular cylinder whose axis is line b and radius r. So it is the intersection of line d with the cylinder. (See figure 7)

If line d intersects the cylinder, the problem has 2 geometric solutions.

If line d is tangent to the cylinder then the problem has a geometric solution.

If line d is neither tangent to nor intersects the right circle cylinder then the problem has no solution.
**Figure 7: The intersection of a line and a cylinder**

Project both lines a and d onto the $\pi_2$ in the direction of projection which is line b. The new projection of b is a point. The new projection of the cylinder is a circle. Its center is the new projection of b. This circle has radius r. The new projection of line d is a straight line d'.

If d' intersects the circle, the problem has 2 geometric solutions.

If d' is tangent to the circle, the problem has 1 solution.

If line d’ is neither tangent to nor intersects the circle then the problem has no solution. (See figure 8)

**Problem 5:** Given three skew lines a, b and c. Draw a straight line t intersecting the above three lines at 3 points A, B, C so that AB=AC.

To solve this problem, we project all three lines onto the $P_g2$ in the direction of line a. We obtained a new projection.

Draw a straight line passing through A’ parallel to b’, intersecting c’ at E’. Line c’ intersects line b’ at point E’. Draw a half circle with center O’ passing through E’ cutting b’ at C’. Join C’ to A’, cut b’ at B’. Then project back to the original projection (see figure 9).

**Figure 9: A line intersects 3 skew lines**

**Problem 6:** Given two skew lines a and b.

Draw the horizontal line MN, M∈a, N∈b, MN=d (see figure 10).

**Figure 10: Draw horizontal line MN**
We project two lines a and b onto the Pg 2 in the direction of line b. We obtained a new projection. Projection of line b is a point b’2 and projection of line a is line a’2. Draw a circle with center b’2 and radius equal to d, cutting a’2 at M’2. From M’2 we infer M2. From M2, infer M1. Draw a line passes through point M1 and parallel to x axis. This line intersects b1 at N1.

III. CONCLUSION

Geometry is a science that is widely applied in engineering and life. It is very complicated and there are many ways to solve it. To solve geometric problems, we must think for ourselves and find solutions. So writing articles about this subject is very difficult. Because it has no experiments, there are no cited experimental data. This article presents a new solution to solve some geometric problems. It helps us solve faster and easier to understand.

REFERENCES


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